Swapskills Pte Ltd

Duration and Interest Rate Risk

Rates

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Yield curves represent the average yield demanded by all capital markets investors for a particular credit quality and a particular maturity. The par (yield) curve represents the yield to maturity on bonds trading at (or very near) their par values.

The spot curve or the zero coupon curve represents the discount rates for cash flows to an investment at different points of time into the future and is constructed from the par curve by bootstrapping.

The forward curve is derived from the spot curve and represents the lending rate for a certain time frame beginning at a point in time into the future.

The swap curve is the same as the par yield curve but is derived differently for published indices such as LIBOR. It is derived as equivalent fixed rate you would pay against receiving floating payments on the index which is expected to move in line with the forward curve.

What is Interest Rate Risk?

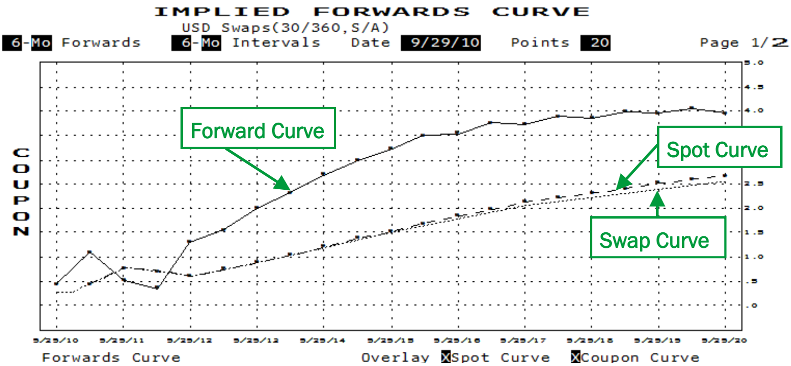
As deals happen in the market place, the shape of the yield curves change. The risk that value of your current bond or swaps portfolio will fall in response to rising rates is called Interest Rate Risk.

Inside this issue:

Points of interest

- **A bond's inherent interest rate risk depends on it's coupon & maturity, embedded options and the market yield.**
- **I.R. Risk can be measured using a complete valuation of using duration and convexity approximations**
- **Floating Rate Bonds have minimal interest rate risk**
- **PV01 is a popular market risk measure for swaps**

The Yield Curves—Revisited



Factors affecting interest rate risk

HIGHER COUPON: implies more value in the form of fixed interest payments is received prior to bond maturity, and a change in interest rates has a lesser effect on bond value. Hence, interest rate risk is lower.

LONGER MATURITY: implies more time for uncertainty over rates. This uncertainty means that large changes in interest rates and the shape of the yield curve, can significantly affect bond value. Hence, interest rate risk is higher.

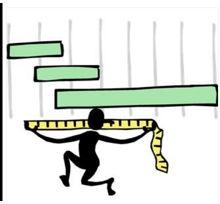
HIGHER MARKET YIELD: means that cash flows into the future are discounted at a higher rate and the PV for a marginal dollar received into the future is lesser and hence the effect on bond price is lesser. Hence interest rate risk is lower.

EMBEDDED CALL: When rates decline, the bond price will not increase above the call price, as issuers will choose to refinance at lower rates. This means that for a bond with an embedded call, interest rate risk is lower than that of an option free bond.

EMBEDDED PUT: When rates rise, the bond price will not rise above the put price, as investors will choose to reinvest at higher rates. This means that for a bond with an embedded put, interest rate risk is lower than that of an option free bond.

IN SUMMARY:

Higher Coupon → Lower Interest Rate Risk
Longer Maturity → Higher Interest Rate Risk
Higher Market Yield → Lower Interest Rate Risk
Embedded Call → Lower Interest Rate Risk
Embedded Put → Lower Interest Rate Risk



How do we measure interest rate risk?

Consider the following two bonds–

Bond	Coupon	Maturity	Price	YTM	
A	7% annual	5 yrs	$104.21	6%	FV=100, PMT=7, I/Y=6, N=5, PV=?
B	6% semi annual	10 yrs	$86.41	8%	FV=100, PMT=3, I/Y=4, N=20, PV=?

Let us build a portfolio combining a $10 million face-value position in each of them and study the change in the portfolio value for shifts in the yield curve.

1 CURRENTLY

Bond	Notional	Price	YTM	
A	$10,000,000	$10,421,000	6%	FV=100, PMT=7, I/Y=6, N=5, PV=?
B	$10,000,000	$8,641,000	8%	FV=100, PMT=3, I/Y=4, N=20, PV=?
Portfolio Value		$19,062,000		

2 IF THE YIELD CURVE SHIFTS PARALLELLY BY +50bp

Bond	Notional	Price	YTM	
A	$10,000,000	$10,208,000	6.5%	FV=100, PMT=7, I/Y=6.5, N=5, PV=?
B	$10,000,000	$8,338,000	8.5%	FV=100, PMT=3, I/Y=4.25, N=20, PV=?
Portfolio Value		$18,546,000		The portfolio reduces 2.707% in value

3 IF THE YIELD CURVE SHIFTS PARALLELLY BY +100bp

Bond	Notional	Price (%)	YTM	
A	$10,000,000	$10,000,000	7%	FV=100, PMT=7, I/Y=7, N=5, PV=?
B	$10,000,000	$8,049,000	9%	FV=100, PMT=3, I/Y=4.5, N=20, PV=?
Portfolio Value		$18,049,000		The portfolio reduces 5.314% in value

On an individual basis, the value of bond A changes more for a 1% change in yield. This can be attributed to a higher coupon rate (as maturity and market yields are similar). Hence we can say ***Bond A has greater interest rate risk***!

This process of re-evaluating your bond portfolio to market is called marking the portfolio to market.

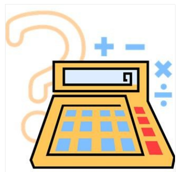
Duration and Convexity

Our earlier periodic re-evaluation of a bond portfolio to assess our interest rate risk is sometimes called a "*Complete Valuation*" or a "*Scenario Analysis*". However scenario analysis can be complex for a portfolio comprising several bonds, and even more so if the bonds have complex structures (callable, convertible, embedded put, sinking fund or amortization provision).

However by making a simplifying assumption of strictly parallel yield curve shifts, we can make the estimation of interest rate risk simpler using two measures called "**Duration**" and "**Convexity**" which will be introduced later.

Duration is a measure of interest rate sensitivity and is used synonymously with the term "Interest Rate Risk"

Computing Duration and Dollar Duration



Duration is the percentage change in the price of a bond for a 1% change in yield. This however is an approximation we make for simplicity as we will see later.

$$Duration = -\frac{\%\Delta Vb}{\%\Delta y} = -\frac{\frac{Vb1 - Vb0}{Vb0}}{y1\% - y0\%} = -\frac{percentage\ change\ in\ bond\ price}{yield\ change\ in\ percent}$$

Dollar duration of a bond is defined as the change in bond price in dollar terms in response to a 1% change in yield.

Example

The yield on the bond increases from 8% to 10%, and the duration of the bond is 6. Calculate the current bond price if the bond traded at par at the start of the period.
Answer: % change in bond price = - (10%-8%) * 6 = - 12%. The bond price falls by 12%. The bond hence trades at $88 per $100 par.

Example

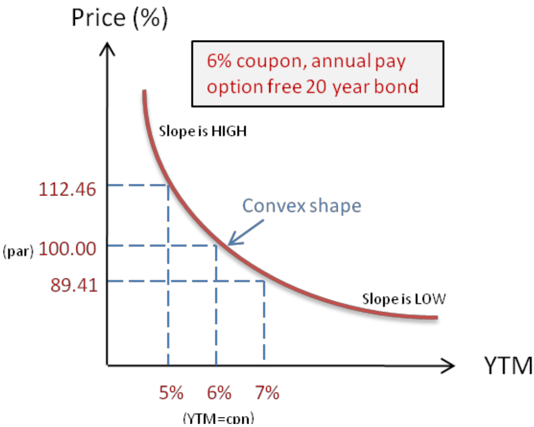
The price of a bond decreases from $92 to $89 (per $100 par) in response to a rise in yield from 8% to 10%. Calculate the dollar duration of your position if you hold a $1.7 notional position in this bond.

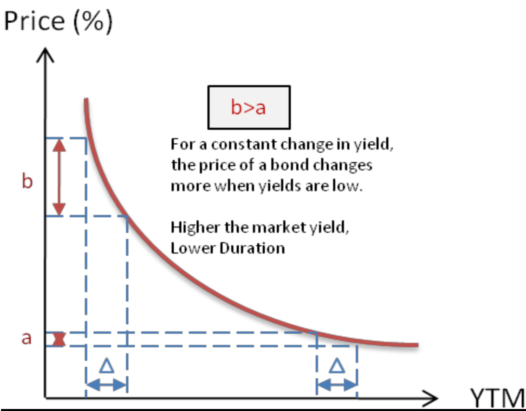
$$Duration = -\frac{89\% - 92\%}{10\% - 8\%} = 1.5$$

Answer: For a 1% rise in yield, bond price rises 1.5%.

The Price v YTM Profile for an Option free bond

We know that as the yield increases, the discount rate for interest and principal into the future of the bond increases and hence bond price decreases. The exactly profile of this relationship for an option free bond is as below—





- An option free bond has positive convexity, i.e the bond's price falls at a decreasing rate as yields rise.
- Since the price rise is greater than the price fall for a unit change in yields, convexity is advantageous to the bondholder.
- Duration of a bond at any yield is defined as the instantaneous rate of change of price with respect to yield (i.e. the slope of the Price v YTM profile).

$$Duration = -\frac{dVb}{dYTM}$$

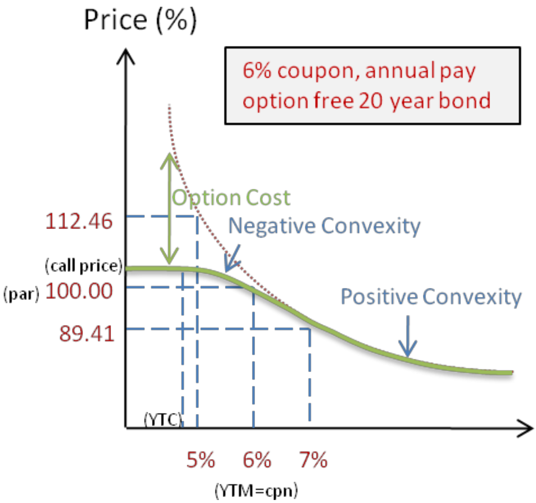
The Duration Approximation

$$Duration = -\frac{dVb}{dYTM} \approx -\frac{\Delta Vb}{\Delta YTM}$$

<u>Remember this:</u>

This approximation holds good only for very small changes in yields. We shall take a look at this later!

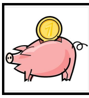
The Price v YTM Profile for a Callable Bond



The Price v YTM profile for a callable or pre-payable bond

- For callable bonds, when rates fall below a certain level, the principal is redeemed using the call provision as the issuer would like to refinance at lower rates.
- For a pre-payable bond the principal is refunded using the prepayment provision or the bond. New debt is issued at lower rates and the proceeds are used to refund the pre-payable bonds.

- As a result there is price compression i.e. a limit to the bond's price appreciation in response to decreasing yields.
- At these lower yields, callable and pre-payable bonds exhibit negative convexity.
- At higher yields the price volatility of a callable bond is similar to that of an option free bond, however at low yields, the price volatility of callable bonds is lower than that of option free bonds.
- The duration is simply the slope of this curve. We can clearly see that the slope decreases at lower yields for callable bonds, and hence the interest rate risk decreases.
- The difference in price between the option free bond and the callable bond when they diverge at lower yields is the value of the embedded call option on the bond.

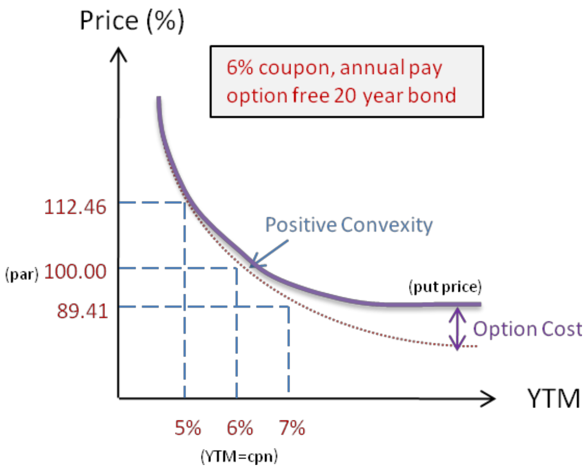


When the bond is called or prepaid, the principal must be re-invested at lower rates. This risk to the investor is called **"Reinvestment Risk"**

The Price v YTM Profile for a Puttable Bond

- For puttable bonds, at a higher yields bond price falls only as much as the put price determined by the embedded option in the contract.

- For a convertible bond the bond value cannot fall below the price of common stock. If yields rise, and bond price falls steeply, investors will exercise the convertible option.



The Price v YTM profile for a puttable or convertible bond

- As a result there is a limit to the bond's price depreciation in response to increasing yields.

- These bonds still exhibit positive convexity and similar price volatility to option free bonds at low yields. However at higher yields, the price volatility is much less than that of the option free bond.

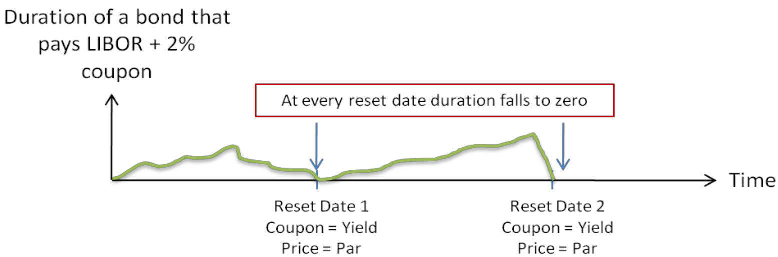
- The duration is simply the slope of this curve. We can clearly see that the slope decreases at higher yields for puttable bonds, and hence the interest rate risk decreases.

- The difference between the price of a puttable / convertible bond and the otherwise similar option free bond when their prices diverge at higher yields is the value of the embedded put option on the bond.

SNIPPET BOX:

Interest Rate Risk of a Floating Rate Bond

For a floating rate security, the coupon rate is reset to the yield as determined by the market periodically. This means that the floating rate bond will trade at par or very near to par at all times, and will hence have a low price volatility i.e. a low duration.



Lesser interest rate risk is however the main reason for floating rates in the first place! Floating rates ensure that lending occurs in the bond market at rates that are representative of yields demanded by investors.

Between coupon reset dates, bonds may trade away from par due to differing expectations on yield in the marketplace. However the market's view is factored into the coupon at every subsequent reset date and the bond value returns to par.

When does the bond value not return to par on the coupon reset date?

A There is an embedded cap / floor in the bond that set's a minimum or maximum bound on the coupon that causes the coupon to vary from the market yield.

B The spread over LIBOR is determined at issuance (Say LIBOR+2%). However since issuance perceptions of credit quality and liquidity of the bond has changed in the marketplace and the yield demanded now represents a higher or lower spread than the fixed 2%

SNIPPET BOX:

Different Estimates for Duration

Macaulay Duration

— The measure was first proposed by Fredrik Macaulay in 1938.

— It describes the bond's price sensitivity to interest rate changes with a single number.

— It is a measure of the estimated amount of time (in years) a bondholder must wait to recover the true cost of the bond.

— It can be perceived as a ***cash flow centre of gravity.***

FORMULA

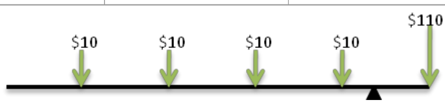
$$D = \sum_{i=1}^{N} T_i \left[\frac{C_i}{B(1+R)^{T_i}} \right]$$

EXAMPLES

For a zero coupon bond: Consider a 5 year maturity zero coupon bond. It has only one cash flow in 5 year's time. Hence it's Macaulay Duration is 5. We can hence say in response to a 1% increase in yield the value of the zero coupon bond will fall by 5%

For a coupon bond: Consider a 5 year maturity 10% coupon bond that yields 11%. The Macaulay Duration is computed as below—

Time (i)	Cashflow	PV (11% Ann)	Weight ($PV_i/\Sigma PV$)	Weight x Time
1	10 Dollars	9.0090 Dollars	9.35%	0.09 periods
2	10 Dollars	8.1162 Dollars	8.43%	0.17 periods
3	10 Dollars	7.3119 Dollars	7.59%	0.23 periods
4	10 Dollars	6.5873 Dollars	6.84%	0.27 periods
5	110 Dollars	65.2796 Dollars	67.78%	3.39 periods
Total	150 Dollars	96.3041 Dollars		4.15 periods
		Macaulay Duration =		4.15 years



Modified Duration

— The measure was first proposed by Hicks in 1939.

— It is derived from the Macaulay Duration but accounts for the current YTM

— It is the Macaulay Duration discounted by the 1 period yield

FORMULA

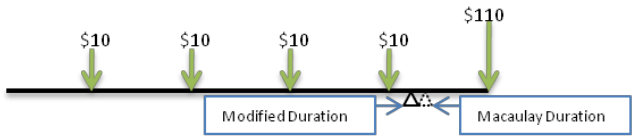
$$Modified\ Duration = \frac{Macaulay\ Duration}{1+\frac{YTM}{n}},\ n = no\ of\ coupon\ periods\ per\ year$$

EXAMPLES

For a zero coupon bond: Modified Duration is not defined for a zero-coupon bond. The duration of a zero-coupon bond is equal to it's maturity.

For a coupon bond: Consider a 5 year maturity 10% coupon bond that yields 11%. The Macaulay Duration was computed as 4.15 years.

$$Modified\ Duration = \frac{4.15}{1+\frac{0.11}{1}} = 4.1048$$



Disadvantages of Macaulay and Modified Duration

Macaulay and Modified Duration are measures that are based on expected cash-flows for an option-free investment. They aren't suitable for measuring the interest rate risk of bonds with embedded options as they don't factor in expectations on optional cash-flows.

SNIPPET BOX

Effective Duration

- It is a measure that accounts for the convexity of the Bond Price v YTM relation, and is valid for both option free bonds and bonds with embedded options.
- When the yields decrease, bond price increases and vice versa. However the amount by which bond price increases or decreases for a unit change in yields is not the same due to the convexity of the relation.
- Effective duration better approximates this relation by taking an average of the price rise / fall in response to a one unit fall / rise in yield.

FORMULA

$$Effective\ Duration = \frac{\%\Delta Vb_{1\%\ fall\ in\ yield} + \%\Delta Vb_{1\%\ rise\ in\ yield}}{2}$$

$$= \frac{\frac{Vb_{fall} - Vb_0}{Vb_0 \times \Delta y} + \frac{Vb_0 - Vb_{rise}}{Vb_0 \times \Delta y}}{2}$$

$$= \frac{Vb_{fall} - Vb_{rise}}{2 \times Vb_0 \times \Delta y}$$

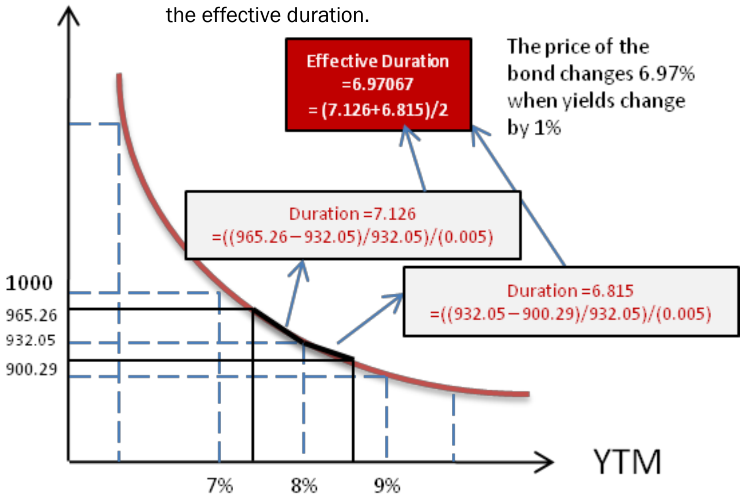
Duration is bond price sensitivity to interest rates

EXAMPLES

Consider a 10 year semi annual pay bond with a 7% coupon and yields 8%. For a $1000 face value it currently trades at $932.05.

Price (%)

For a 50bp rise in yields, the bond price falls to 900.29. For a 50bp fall in yields, the bond price rises to 965.26. Calculate the effective duration.



Note on Effective Duration

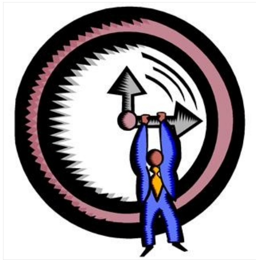
Earlier we computed the Effective duration of the bond using a 50bp rise and fall in yields. If we were to calculate it with a 25bp yield instead—

$$Dur_{fall} = duration\ computed\ from\ a\ 1\%\ fall\ in\ yield = \frac{\frac{948.47 - 932.05}{932.05}}{0.0025} = 7.0468$$

$$Dur_{rise} = duration\ computed\ from\ a\ 1\%\ rise\ in\ yield = \frac{\frac{932.05 - 915.99}{932.05}}{0.0025} = 6.8923$$

$$Effective\ Duration = \frac{7.0468 + 6.8923}{2} = 6.9696$$

We can notice that Dur_{rise} and Dur_{fall} are much closer in magnitude to the effective duration in this case. This shows that the approximation becomes better for smaller changes in yield. As the change in yield falls Dur_{rise} and Dur_{fall} converge towards the effective duration and the estimate of effective duration becomes better. Take a look—



Δy (in bp)	50	25	20	10
Vb (fall)	965.26	948.47	945.15	938.57
Vb0	932.05	932.05	932.05	932.05
Vb (rise)	900.29	915.99	919.18	925.58
Dur (fall)	7.1262	7.0468	7.0275	6.9953
Dur (rise)	6.8151	6.8923	6.9041	6.9417
Effective Duration	6.9707	6.9696	6.9658	6.9685

Effective Duration for Bonds with Embedded Options

The effective duration measure takes into account the convexity of the price v yield relation. The promised cash-flows to the bond take into account the adjustments required for embedded options as these are reflected in the price v yield relation. Hence Effective duration is a good measure to estimate the interest rate risk of bonds with embedded options!

Duration of a Portfolio

The Duration of a Portfolio is the weighted average of the duration of the constituent bonds.

Example

Bond	Market Value	Weight	Duration	Weighted Duration
A	1,000	0.1	6.7	0.67
B	3,000	0.3	7.3	2.19
C	6,000	0.6	5.4	3.24
Value	10,000	Portfolio Duration		6.1

Interpretation of Portfolio Duration

When yields were to change by 1%, the value of the bond will change by 6.1% in the other direction.

Limitations of Portfolio Duration

What is the bonds in the portfolio had the following characteristics—

- **A** Callable, 1 year maturity, AAA credit rating
- **B** Bullet Bond, 2 year maturity, AA credit rating
- **C** Bullet Bond, 10 year maturity, AA– credit rating

It is possible that the market expects a greater yield on longer term bonds than short term bonds, or expects a greater spread over treasuries for AA– rated bonds over time, or expects a greater yield for callable bonds. Hence it is possible that yields on certain bonds in the portfolio change selectively.

For example if the yield curve were to steepen, yields on bond C increase (10 yr maturity) while yields on bonds A and B remain the same. Hence the value of portfolio changes purely out of a fall in the market value of bond C!

So, portfolio duration is a good measure only when a 1% change in yield on the portfolio implies a 1% change in yield on each constituent bond, i.e. it is a good measure only for parallel yield curve shifts!

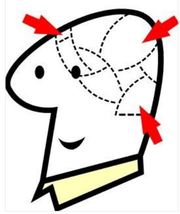
Convexity

Macaulay and Modified Duration are measures that are based on expected cash-flows for an option-free investment. They aren't suitable for measuring the interest rate risk of bonds with embedded options as they don't factor in expectations on optional cash-flows.

PV01

Macaulay and Modified Duration are measures that are based on expected cash-flows for an option-free investment. They aren't suitable for measuring the interest rate risk of bonds with embedded options as they don't factor in expectations on optional cash-flows.

Time to bring out the grey matter



1. A bond currently trades for $10783.20 and has a yield of 7.99%. The duration of the bond is 7.5. if the yield were to rise to 8.29% what will be the new price of the bond?

Answers

1. The change in yield = 8.29% - 7.99% = 0.3%
   Percentage fall in price = 7.5 * 0.3% = 2.25%
   Hence every par dollar is now worth $(1-0.0225) = $0.9775
   Hence the new market price = 0.9775 * 10783.20 = $10540.578

2.

Answers (contd..)

1. A bond currently trades for $10783.20 and has a yield of 7.99%. The duration of the bond is 7.5. if the yield were to rise to 8.29% what will be the new price of the bond?

Notes

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