Swapskills Pte Ltd

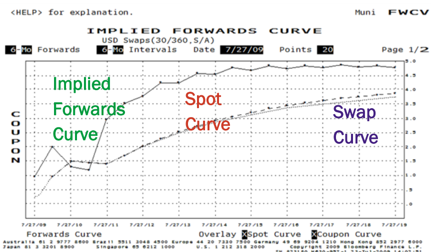
Yield Curves and Yield Spreads

Rates

102

Corporations and Financial Institutions participate in the capital markets to raise funds to fund their projects and operations. At any point in time, investors in the capital markets demand a specified yield for dispensing with their capital. The yield demanded by investors varies depending upon the term for which the funds are lent out (tenor), the servicing power of the party borrowing the funds (credit quality) and the ability to recover the funds in times of need based on the negotiability of the funding instrument (liquidity) amidst others.

What is a Yield Curve?



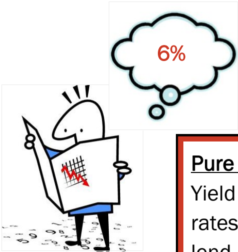
The plot of yields versus term to maturity is called a yield curve. The simplest most frequently used yield curve plot is the Treasury yield curve. The Treasury spot yield curve shows the yields for US Treasury securities (bills, notes and bonds) with maturities from 3 months to 30 years.
The forwards curve, spot curve and the swap curve shown above are explained later.

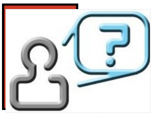
Inside this issue:

Points of Interest

- **Yields curves can be upward or downward sloping**
- **Spreads may be added to the nominal Treasury yield for credit quality, liquidity, issue size, embedded options, taxability amongst others**
- **The forward curve is derived from the spot curve**
- **The swap curve is fixed rate that shall be quoted against floating payments receivable on the index in subject**

Theories on Yield Curves— What Drives Investor Expectations?





Pure Expectations:

Yield for a particular maturity is an average of future short term rates. This means that it must be indifferent for an investor to lend his money for 6 months, or otherwise for 3 months and then again lend out the principal and interest for another 3 months. If short term rates rise, this will necessarily be reflected in a higher longer term rate. *(E.g. If the 3 month lending rate 3 months from now were expected to increase, the present 6 month rate would increase correspondingly and consequently exceed the current 3 month lending rate)*

A Preference for Liquidity:

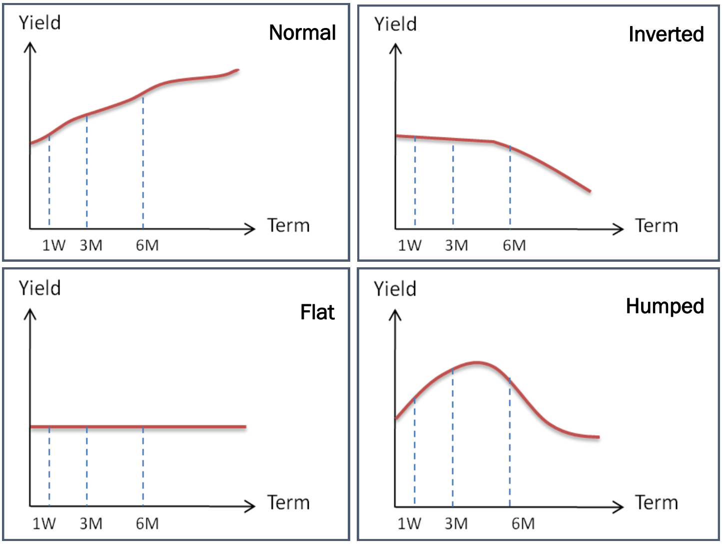
In addition to volatility of future short term rates, investors require additional compensation for the lack of liquidity for lending funds in the long term. Hence the long term rates equal the short term rates + a liquidity premium (compensation demanded based on investors' liquidity preference).

Market Segmentation:

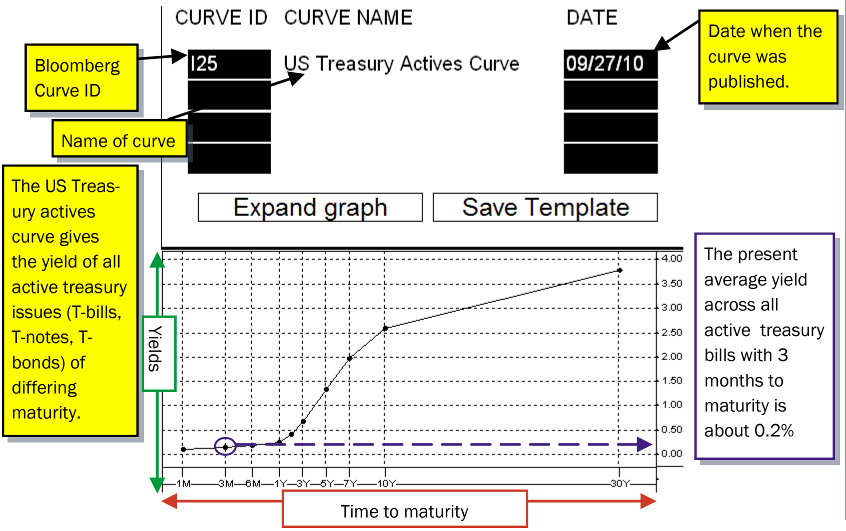
This theory suggests that investors and borrowers have a preferred investment / financing habitat i.e. they prefer to lend / borrow over a certain maturity range. Retail borrowers (such as credit card holders) have a short term borrowing horizon. Institutional investors prefer holding assets whose maturity matches their liabilities. Here it is the *supply and demand* for funds that drives the yields. However when rates aren't lucrative for investment, investors may choose another market segment to invest in, or change their habitat.

Yield Curve Shapes

Yield curves can take any shape, but four shapes are commonly encountered—



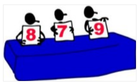
Interpreting a yield curve



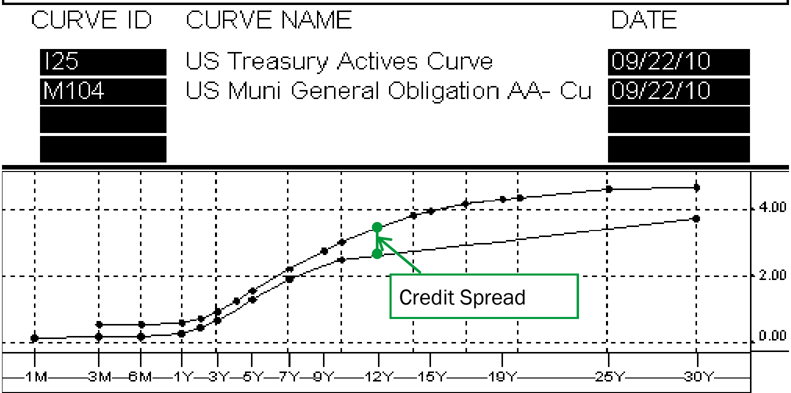
Yield Spreads

A spread is a premium to be added to the risk free yield (Treasury rate) demanded by US Treasury investors. An increased yield may result from several reasons—

Credit (Quality) Spread:



It is the difference between the yield of two issues that are similar in all respects except for their credit rating. Example, the difference between the yields of a 3m maturity AA rated general obligation municipal bond and a A rated General Obligation (GO) muni. They reflect the added gain from investing in an issue of poorer credit quality.



Curve Descriptions

I25—US Treasury Actives Curve	*M104—US Muni General Obligation AA– Curve*
This shows the average yield for US Treasuries. US Treasuries have a AAA rating are backed by the US Govt.	The curve shows the average yield for US GO Munis with an average AA– rating from S&P and Moody's.

For a similar term and liquidity, US Treasuries and US GO Munis differ in credit quality. The Difference in yields is purely a credit spread as shown!

Variation of Credit Spread:

In healthy economic conditions, corporations have more steady cash flows and can service debt better. So, credit spreads tighten! Conversely, in a contracting economy credit spreads widen.

Alternatively, from a supply and demand perspective, during an economic contraction investors prefer higher-quality issues. As a result, demand for lower grade issues falls raising their yields.

Liquidity Spread:

Investors prefer more liquidity to less liquidity and will pay a premium for added liquidity. The less liquid the issue the greater the yield demanded.

Higher the issue size, the more active the secondary market in the issue and hence greater liquidity. So, these issues have a lower spread over Treasuries.

CURVE ID	CURVE NAME	DATE
I25	US Treasury Actives Curve	09/22/10
F936	EUR US AAA BFV Curve	09/22/10

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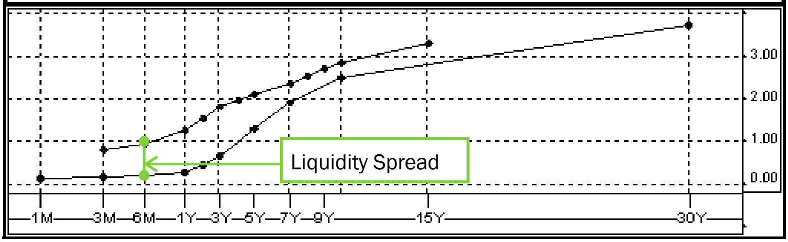


Curve Descriptions

I25—US Treasury Actives Curve

This shows the average yield for US Treasuries. US Treasuries have a AAA rating are backed by the US Govt.

F936—EUR US AAA BFV Curve

The curve is populated with EURO denominated fixed rate bonds sold by US issuers with an average AAA rating issued by S&P, Moody's , Fitch or DBRS.

The curve above shows the US Treasury spot curve and the spot yield curve for EUR denominated AAA bond sector of the US bond market.

Despite similar credit quality increased yields for EUR bonds of a similar maturity is due to lesser liquidity.

The larger the size of the issue, the more active the secondary market in the issue, greater the liquidity!

Options Spread:

A bond investor will require an added yield for holding a callable bond, as the bond may be called when conditions are favourable for the issuer. Hence, the spread to Treasuries will be higher than a similar option-free issue. Yield spreads are similarly higher for bonds with sinking fund provisions / other pre-payment provisions.

Conversely, for a puttable bond or a convertible bond, the spread over Treasuries, as compared to a similar option-free bond, will be lower.

It is 3rd October 2010 and you are currently holding the following bonds on one of your books. They all mature in 2015. Currently, the yield to maturity for a 5 year T-Bond maturity bond is 1.2%. Compare the yields and decipher option spreads on comparable callable / puttable and option free issues.

Bond Name	Notional (USD '000s)	Coupon	Maturity	Rating	Yield	Structure
GMAC	8,200	5% s.a	2015	AA	2.3%	BULLET
DCX	1,300	5% s.a	2015	BB	4.0%	BULLET
FIAT	8,000	4% s.a	2015	AA	2.8%	CALLABLE
DCX	9,000	4% s.a	2015	BB	4.4%	CALLABLE
FIAT	5,000	3.5% s.a	2015	AA	2.3%	BULLET
VLVY	3,300	4% s.a	2015	AA-	3.1%	CONVERTIBLE
F	4,520	6% s.a	2015	BBB	4.6%	BULLET
VLVY	3,200	3.5% s.a	2015	AA-	3.3%	BULLET
GMAC	5,200	3.5% s.a	2015	AA	1.9%	PUTTABLE
F	1,100	3% s.a	2015	BBB	4.7%	BULLET

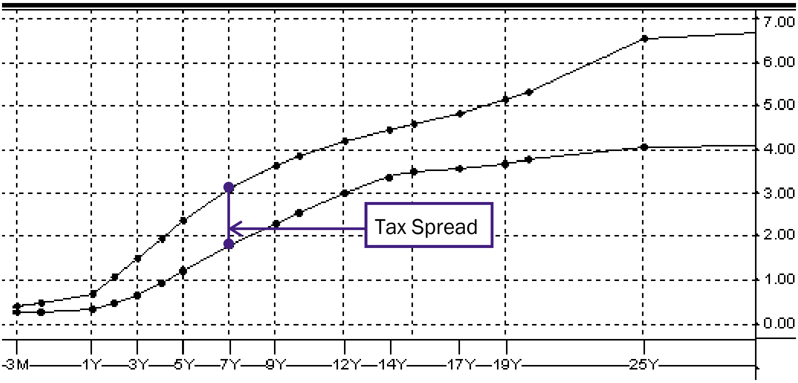
The bonds have a comparable credit quality and maturity, similar liquidity (issue size) and tax properties. The difference in yield is due to the embedded call. Call Spread = 0.5%

The bonds have a comparable credit quality and maturity, similar liquidity (issue size) and tax properties. The difference in yield is due to the embedded put. Put spread = -0.4%

Tax Spread:

Investors expected to be compensated with a premium for taxable issues, as tax payments on interest and capital gains decrease their holding period return. For example a AA rated municipal tax free issue will have a lower spread over Treasuries than a AA rated corporate taxable bond.

CURVE ID	CURVE NAME	DATE
M49	US Muni General Obligation A	09/27/10
M901	US Muni Taxable AAA Curve	09/27/10



Curve Descriptions

M49—US Muni General Obligation AAA Curve

The curve is populated with US municipal general obligations (G.O.) with an average rating of AAA from Moody's and S&P.

M901—US Muni Taxable AAA Curve

The curve is populated with taxable US municipal general obligation (G.O.) and revenue bonds with an average rating of AAA from Moody's and S&P.

Interest received from holding G.O. Munis are generally tax-exempt. Municipalities issue tax exempt bonds to appeal to investors who prefer tax benefits over added returns. The two curves shown above are identical in credit rating. For any particular maturity they vary only in the taxable nature of their coupons and capital gains. Hence the area between the 2 curves represents the tax spread!

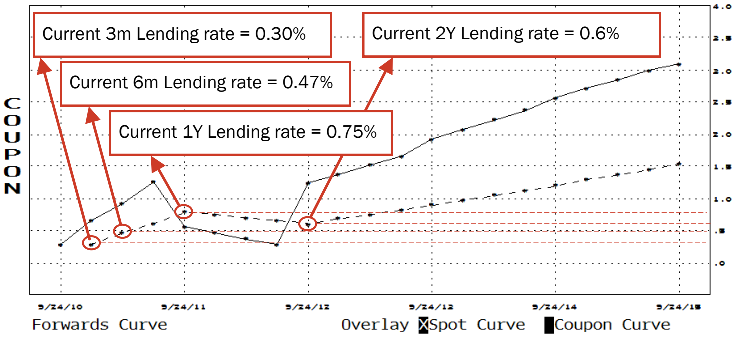
LIBOR—a benchmark for floating rate debt

LIBOR (London InterBank Offered Rate). It is the average rate at which banks will lend funds to each other for maturities ranging from overnight to 12 months, and 10 different world currencies. Every morning a select list of banks are asked the question, "*At what rate could you borrow funds, were you to do so by asking for and accepting inter-bank offers in a reasonable market size just prior to 11 am?*". The answers from the various banks are then collated, averaged and then published by the British Bankers' Association as LIBOR for different maturities and currencies.

LIBOR is the primary benchmark for short term interest rates globally. Around $350 trillion of swaps and $10 trillion of loans are indexed to LIBOR.
It is important to note that LIBOR represents inter-bank risk. It represents the lowest real-world cost of funding in the London market. Loans to corporates are made at a small spread over LIBOR based on credit quality.

Spot Rate

A spot rate for a particular tenor is the yield demanded by an investor to lend out his capital for the specified period of time. Conceptually spot rates may be viewed as the discount rates for zero coupon bonds with a maturity equal to the term of the subject investment. The plot of spot rates versus their term to maturity is called a "Spot Curve" or the "Zero Coupon Curve".

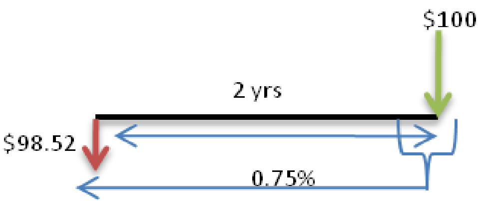


The dotted line above shows the USD Libor spot curve. The different points on the spot curve correspond to lending rates for different tenors as illustrated.
In valuing bonds / swaps / other securities where cash flows happen in the future, these cash flows must be discounted using the spot rate, as this will account for the reinvestment of the coupon received from holding the security.

Valuing a zero-coupon bond from the spot curve

Evaluate the price of a 2 year maturity zero coupon bond with a $100 notional using the spot curve on the previous page.

<u>Answer:</u>



From the spot curve we see that cash flows 2 years into the future must be discounted at the spot rate of **0.75%**. We can hence evaluate the price of a zero coupon bond as—

$$Vb = \frac{100}{(1+0.0075)^2} = \$98.52$$

FV = 100, PMT = 0, N = 2, I/y = 0.75
CPT PV; PV = 98.52

Valuing a coupon bond from the spot curve

Evaluate the price of a 2 year maturity $1000 notional 7% coupon bond, payable semi-annually using the spot curve shown on the previous page.

Cash Flow	Time	Spot Rate	Discount Factor	Discount Factor	PV
35	0.5 years	0.47%	$\frac{1}{\left(1+\frac{0.0047}{2}\right)^1}$	0.99765551	34.9179428
35	1 years	0.75%	$\frac{1}{\left(1+\frac{0.0075}{2}\right)^2}$	0.99254197	34.7389692
35	1.5 years	0.69%	$\frac{1}{\left(1+\frac{0.0069}{2}\right)^3}$	0.98972100	34.6402352
1035	2 years	0.60%	$\frac{1}{\left(1+\frac{0.0060}{2}\right)^4}$	0.98808946	1022.67259

Value of bond = $1126.97

Cashflow Schedule

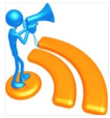
The Table above that shows the payments received from an investment, their discount factors, and their present values, is called a "Cashflow Schedule."

Sources of income / yield on a bond

Periodic coupon payments (interest)	Capital gains / losses from price rise / fall	Reinvestment of coupon payments

Traditional Bond Yield Measures

Current Yield



This is the most simple yield measure for a bond and is computed as below—

$$Current\ yield = \frac{\text{"Annual" coupon payment}}{price\ of\ bond}$$

This considers only the periodic interest income on the bond and ignores the other 2 sources of income, price change and reinvestment.
Hence for a 10 year $1000 par, 7% semi-annual coupon bond that currently trades at $846.46, the current yield would be (70/846.46) = **8.27%**

Yield to Maturity (YTM)

The yield to maturity is an internal rate of return. It is the constant discount rate that must be used to discount all of the bond's cash flows such that the value of the bond equals its market price.

In the previous example, we had valued a bond price using spot rates and found its value to be **$1126.97**. Compute its yield to maturity.
<u>Answer</u>
The yield to maturity is computed as follows—

$$1126.97 = \frac{35}{1+\frac{YTM}{2}} + \frac{35}{(1+\frac{YTM}{2})^2} + \frac{35}{(1+\frac{YTM}{2})^3} + \frac{1035}{(1+\frac{YTM}{2})^4}$$

We can solve for this using a financial calculator as below—

FV = 1000, PMT=35, PV=-1126.97, N=4, **CPT I/y**; I/y = 0.3%
YTM = 2 x 0.3% = 0.6%

2 immediate observations—
a) Since the Coupon Rate > YTM, the bond trades at a premium to par
b) YTM is some kind of average of the spot rates used to value the bond earlier.

Example on calculating YTM for annual pay bond

Consider an annual pay 10 year, $1000 par value 5% coupon bond that sells for $840.46. Calculate the annual pay YTM and the current yield.

Answer:

Since this is an annual-pay bond we write—

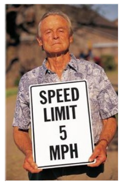
$$840.46 = \sum_{i=1}^{10} \frac{50}{(1+YTM)^i} + \frac{1000}{(1+YTM)^{10}}$$

FV = 1000, PMT=50, PV=-840.46, N=10, **CPT I/y**; I/y = 7.3%

YTM = 1 x 7.3% = 7.3%

Current Yield = 50/840.46 = 5.949%

Limitations of Yield to Maturity



The limitation of the Yield to Maturity measure is that it doesn't account for reinvestment of coupon payments. This forms a significant portion of the compound rate of return realized on a bond.

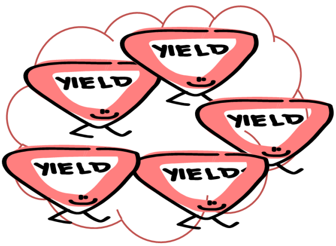
The realized yield on a bond is the actual compound return that was earned on the initial investment. It is usually computed at the end of the investment horizon. For a bond to have a realized yield equal to its Yield to Maturity, all cash flows from the bond must be reinvested at a rate equal to the yield to maturity and the bond must be held until maturity.

Computing the reinvestment income on a bond

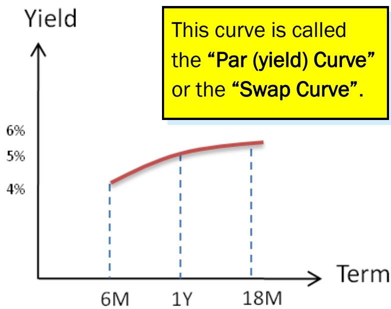
You purchase a 5% coupon, 10 year $1000 par bond. If the YTM is 5% s.a, how much of the bond's yield is accountable to reinvestment of coupon?

Total value in 10 years (compounded at YTM) = $1000(1.025)^{20} = $1638.62
Total coupon income = $25*20 = $500
Principal payment = $1000
Hence reinvestment income = $1638.62—$500—$1000 = **$138.62**
Reinvestment income as a % of total income = 138.62/1638.62 = **8.46%**

BOOTSTRAPPING: Where do spot (zero coupon) rates come from?



At any point in time there are several bonds trading in the marketplace. A par yield curve is compiled by a data service provide like Reuters or Bloomberg, that gives the Yield To Maturity of bonds currently trading around their par values for different maturities. Assume the sample par yield curve is as shown below—

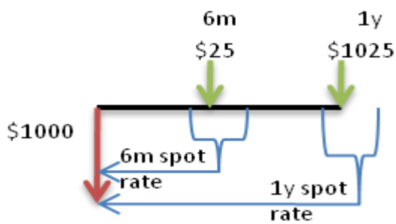


Since the Bonds are trading around par, the yield is in fact equal to the coupon on these issues.
We shall try to construct the spot curve from this par yield curve assuming a $1000 par value. Assume the bonds pay semi-annual coupons.
The process outlined below is called "bootstrapping".

Bootstrapping outlined

A) A bond with 6 months to maturity has a semi annual discount rate of 4% annual or 2.0% semi annual discount rate. We can hence write—
1000 = 1020/(1.02). Hence the semi annual spot rate = 2%, or the annualized 6m spot rate = 4%.

B) Now consider a bond maturing in 1 year. The coupon from the par yield curve is 2.5% semi annual. We discount the cash flows as below—

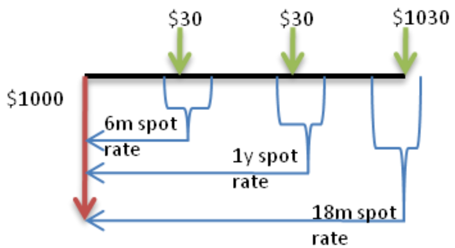


If r is the 1 year spot rate,

$$1000 = \frac{25}{1.02} + \frac{1025}{(1+\frac{r}{2})^2}$$

Solving we get r = 5.0125%

C) Consider a bond maturing in 18 months. The coupon from the par yield curve is 3% semi annual. We discount the cash flows as below—

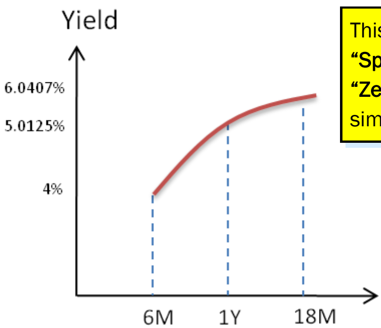


If r is the 18 month spot rate,

$$1000 = \frac{30}{(1+\frac{0.04}{2})} + \frac{30}{(1+\frac{0.050125}{2})^2} + \frac{1030}{(1+\frac{r}{2})^3}$$

Solving we get r = 6.0407%

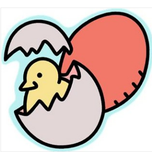
Hence the spot curve looks like this—



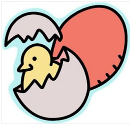
This curve is called the **"Spot Curve"** or the **"Zero Coupon Curve"** or simply the **"Zero Curve"**.

The Spot Curve is constructed by "bootstrapping" the par yield curve

Food for thought!



- Is it the spot rates that are used to price up a bond from which we can compute a bond's Yield to Maturity?
- Or is the bond's Yield to Maturity determined at the point of bond issue, based on which we compute spot rates?
- Sounds like a chicken and egg situation?



Chicken and Egg Explained!

<u>Here's how it works–</u>

When an issuer likes to raise capital he comes to an Investment Bank's Capital Market's Desk.

The pricing suggested is based on the current spot curve on comparables

The Capital Markets team consults yields on comparable issues / strap on an appropriate spread over Treasuries and suggests suitable pricing for the offering.

They extend an offer to bond investors, and the bond is eventually priced and the paper is placed based on bids made by investors. Their expected yield may vary from what the market currently expects.

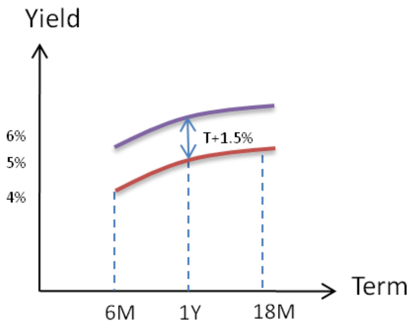
The Offer is underwritten and the deal information goes public.

The data companies collate this to update the par yield curve and hence the spot curve.

Common Spread Measures

Nominal Spread

Consider the Treasury par yield curve (red), and the par yield of a corporate bond (purple) shown below. The nominal spread over Treasuries is a constant yield spread to be added to the Treasuries' par yield curve to obtain the par yield curve of the issue.

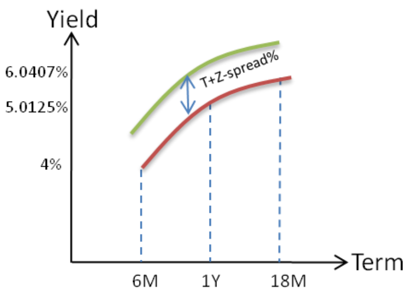


The nominal spread has the same shortcoming as the YTM measure. Discounting all the cash flows at a constant YTM ignores the shape of the spot-yield curve. The use of nominal spreads holds good only when the zero-coupon curve is flat or very nearly flat.

$$Nominal\ spread = YTM_{Bond} - YTM_{Treasury}, for\ the\ same\ maturity$$

Zero Volatility Spread (Z-Spread)

This is the equal amount we must add to each of the rates on the Treasury-spot curve in order to value a risky bond correctly (making its market price equal to the present value of its future cash flows.) This takes into account the shape of the spot curve and hence holds good for all shapes of the Treasury spot curve. It is also called a static spread.

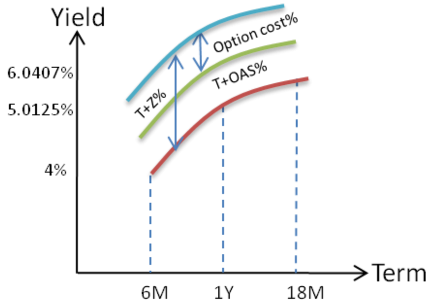


Consider the Treasury spot curve shown to the left. A $100 par value, 5% semi annual coupon corporate bond whose curve is shown in green trades at $89.596. Its Z-spread is calculated by solving the relation below—

$$89.596 = \frac{2.5}{(1+\frac{(0.04+ZS)}{2})^1} + \frac{2.5}{(1+\frac{(0.050125+ZS)}{2})^2} + \frac{102.5}{(1+\frac{(0.060407+ZS)}{2})^3}$$

Solving we get **ZS = 8.90%**

Option Adjusted Spread



Consider the $100 par value, 5% semi annual coupon bond from the previous example on Z-spreads, except that it is callable. As a result of the embedded call it trades at a significant discount to par and it currently trades at $85.71.

Solving the relation below we get a **Z-spread of 12.1%.** As a result the additional 3.2% compensation from the bond in the previous example is to compensate for the embedded option.

$$85.7100 = \frac{2.5}{(1+\frac{(0.04+ZS)}{2})^1} + \frac{2.5}{(1+\frac{(0.050125+ZS)}{2})^2} + \frac{102.5}{(1+\frac{(0.060407+ZS)}{2})^3}$$

However this 12.1% spread includes the call risk in it. (Call risk is the risk that the bond will be called prior to maturity). Option adjusted spread is a measure that removes the option yield component from the Z-spread, and prices in only the credit risk, interest rate risk and liquidity risk.

We can hence write—

12.1%	-	8.9%	=	3.2%
Z-spread	-	OAS	=	Option Cost

Example on Spreads

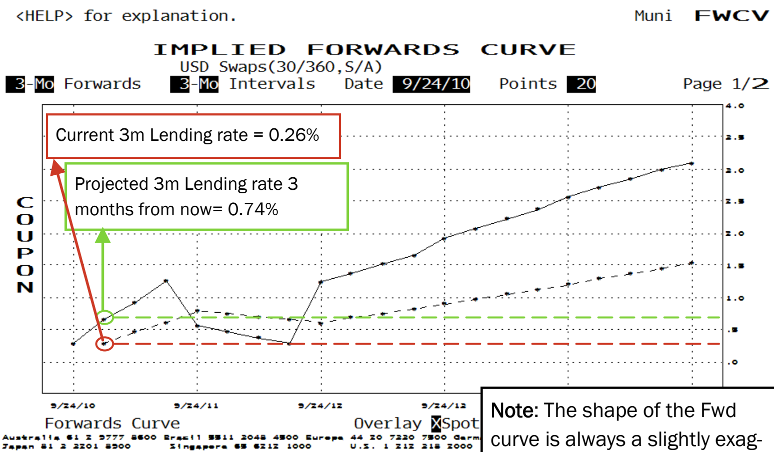
Consider a puttable bond that has a Z-spread of 9%. The option cost is 70bp. What is the Option Adjusted Spread?

Answer: For a puttable bond you pay for the option, hence the option cost is negative.

We know that OAS = Z-spread—Option cost, i.e.
OAS = 9% - (-0.7%) = 9.7%

Hence for puttable bonds, The option adjusted spread exceeds the Zero volatility spread!

Implied Forwards Curve



Note: The shape of the Fwd curve is always a slightly exaggerated version of the spot curve

Now we know what the yields shown on a spot curve mean. We know the present 3 month lending rates and we know the current 6 month lending rates. it leads us to a question—

What is the 3 month lending rate 3 months from now?

This is exactly what the forward curve tells us. The forward curve gives us the lending rates at a point in time in the future based on current market expectations. It is constructed from the spot curve as follows—

Deriving a forward curve

Assume current 3 month USD LIBOR = 0.26%
Current 6 month USD LIBOR = 0.5%

Now from the pure expectation theory we know that the following two alternatives must be indifferent to avoid arbitrage—

PLAN A	PLAN B
Invest a dollar for 3 months at 0.26% and then reinvest these proceeds for another 3 months.	Invest a dollar for 6 months at the current 6 month lending rate of 0.5%.

In Plan A, the rate for the second 3 month period must be such that the client has the same cash at the end of 6 months, whether he opted to invest in plan A or plan B.

Hence we can write—

$$1 * \left(1+\frac{0.0026}{4}\right)\left(1+\frac{{}_{3}r_{6}}{4}\right) = 1 * \left(1+\frac{0.005}{4}\right)^{2}$$

Known 3 month rate

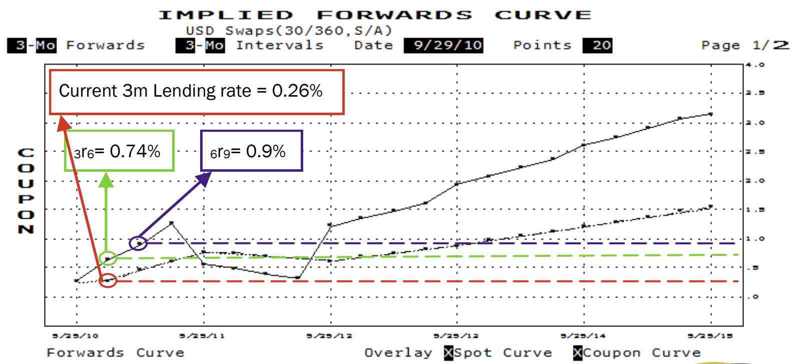
The unknown 3 month rate starting 3 months from today and ending 6 months from today

Known 6 month rate

This gives us ${}_{3}r_{6}$ = 0.7401%

Similarly we can calculate 3 month forward rates at times 6m, 1Y, 2Y etc. from today to construct 3m forward curve. The implied forward curve on 3m LIBOR is shown below.

3 MONTH LIBOR FORWARD CURVE

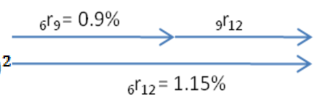


Example

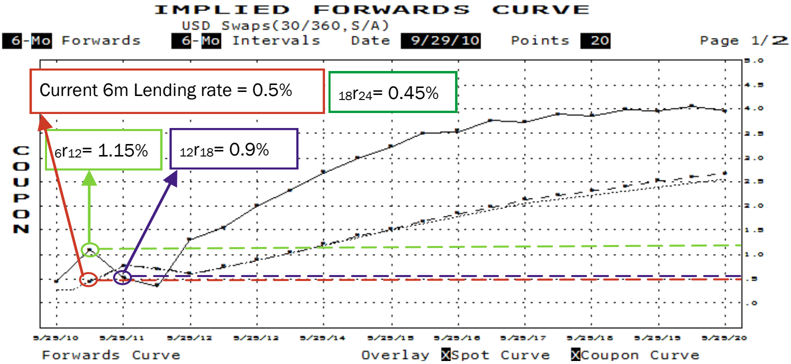
Calculate ${}_{9}r_{12}$ given that ${}_{6}r_{9}$ = 0.9% and ${}_{6}r_{12}$ = 1.15%

Answer: ${}_{9}r_{12}$ = 1.4002%

$$1 * \left(1+\frac{0.0090}{4}\right)\left(1+\frac{{}_{9}r_{12}}{4}\right) = 1 * \left(1+\frac{0.0115}{4}\right)^{2}$$



6 MONTH LIBOR FORWARD CURVE



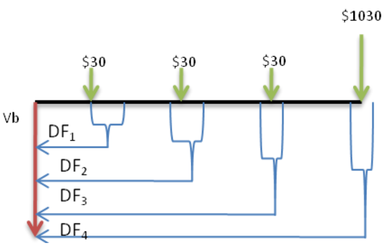
The 6m forward LIBOR curve is similarly constructed by extrapolating 6 month lending rates into the future. For example, ${}_{6}r_{12}$ would denote the 6 month lending rate for a period beginning 6 months from now and ending 12 months from now.

Example

Compute the value of a \$1000 par value, 6% coupon bond which pays semi-annual coupons, that has 2 years to maturity using the 6 month forward curve shown above.

Answer:

We shall discount the bond's cash flows using discount factors as shown below—



$$DF_1 = \frac{1}{1+\frac{0.005}{2}}$$

$$DF_2 = \frac{1}{\left(1+\frac{0.005}{2}\right)\left(1+\frac{0.0115}{2}\right)}$$

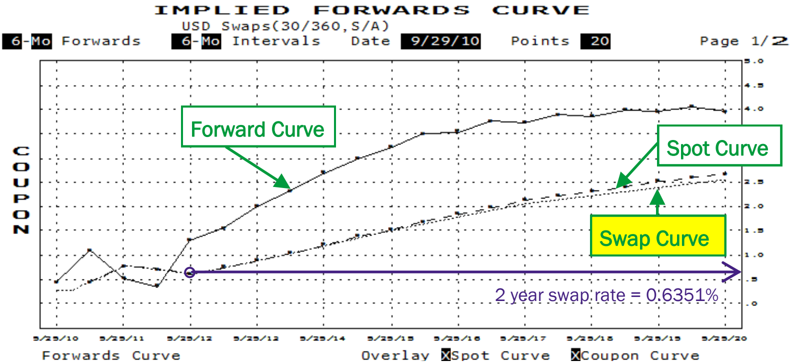
$$DF_3 = \frac{1}{\left(1+\frac{0.005}{2}\right)\left(1+\frac{0.0115}{2}\right)\left(1+\frac{0.009}{2}\right)}$$

$$DF_4 = \frac{1}{\left(1+\frac{0.005}{2}\right)\left(1+\frac{0.0115}{2}\right)\left(1+\frac{0.009}{2}\right)\left(1+\frac{0.0045}{2}\right)}$$

Cash	Time	Discount Factor	PV
30	0.5 y	0.997506234	29.92518703
30	1 y	0.991803365	29.75410095
30	1.5 y	0.987360	29.62080732
1030	2 y	0.985143671	1014.697981

Value of Bond =
Net Present Value =
\$1103.998

Swap Curve



We had earlier seen how to bootstrap a spot curve from a Par (yield) curve, which had also called the Swap Curve. The Par yield curve represented the YTMs on a range of bonds of different maturities which all traded at par (yield = coupon), where the bonds were representative of the risk grade in subject.

We now turn the discussion to the context of the floating index LIBOR.

Question: What would the swap rate represent in this case?

Answer: There are 3 ways to look at the swap curve

(I) Imagine that the banks who make up the relevant LIBOR setting collectively issued a strip of bonds for various maturities ranging from one year to, say, 30 years. If these bonds, representing interbank risk, were priced at par, what would their yield be? This is what is represented by the Par, or Swap curve.

(II) You could also see it as the Risk Free curve (Treasury par yield curve) to which has been added a credit spread. This spread reflects the fact that interbank risk is not as good as the Risk Free rate. This spread is known as the ***Swap Spread***.

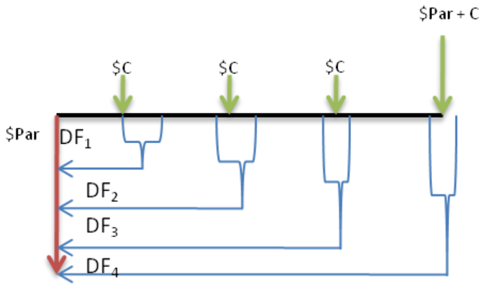
(III) This is the fixed rate I will swap with you over a set period of time. In exchange, you will give me the floating index, such as 6 month LIBOR, at whatever the index rate happens to be on the roll date. We know that the rule of swaps is that the PV of what we pay should equal the PV of what we receive. So if we discount the rates on a forward curve back to a PV, we need to solve for a single fixed rate which has a similar PV to determine the swap rate.

Deriving the Swap Curve from the Forward Curve

The 6 month forward LIBOR rates are as below. Derive the current 2 year swap rate.

Term	6 m Forward Rate
Spot	0.49%
0.5y	1.12%
1y	0.53%
1.5	0.40%

Answer: We shall first derive the Discount factors for payments into the future as below. We divide the rates by 2 as they are semi-annual rates and on a 30/360 basis.



$$DF_1 = \frac{1}{1+\frac{0.0049}{2}}$$

$$DF_2 = \frac{1}{\left(1+\frac{0.0049}{2}\right)\left(1+\frac{0.0112}{2}\right)}$$

$$DF_3 = \frac{1}{\left(1+\frac{0.0049}{2}\right)\left(1+\frac{0.0112}{2}\right)\left(1+\frac{0.0053}{2}\right)}$$

$$DF_4 = \frac{1}{\left(1+\frac{0.0049}{2}\right)\left(1+\frac{0.0112}{2}\right)\left(1+\frac{0.0053}{2}\right)\left(1+\frac{0.0040}{2}\right)}$$

DF1 = 0.997555988

DF2 = 0.992000783

DF3 = 0.989378929

DF4 = 0.987404121

At, issue a floating-rate bond has a value equal to par. This value can change during the life of the bond, but since the floating rate is reset to the market rate at each payment date, the value of the bond will return to par! If we assume a $1000 par value for the floating rate note, and assume the fixed coupon of C—

$$\$1000 = c(DF1) + c(DF2) + c(DF3) + C(DF4) + \$1000(DF4)$$

Solving this we get C = 0.3176%

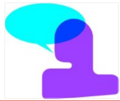
We multiply this by 2 to get the coupon rate on the fixed leg or the two year swap rate = 0.6351%

We can hence see that the fixed swap rate is a weighted average (not a normal weighted average) of the forward rates.

Discussing the market with a client

Theoretically, the forward curve is constructed from the spot curve by bootstrapping. However your client's view may vary from what the technicals indicate. This is where there is opportunity for banks to help a client capitalize on his view which differs from the market's view.

Client says: "I think the forward curve is too steep!"



If that is what you think, then you are saying its PV is too high. Then that must mean that the PV on the fixed rate is also too high. So you should receive fixed (and pay LIBOR) on a swap, since you are receiving *a higher fixed rate* than you think you should be.

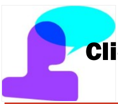
You could also consider selling caps (calls). You collect a premium and, if your view is correct, the LIBOR will not climb so high and the cap will not be exercised.

You could also sell a Forward Rate Agreement (FRA) and benefit if rates stay below the contract level on settlement.

Finally you could sell a payer's swaption, giving a counterparty the right to pay fixed in the future. If you are correct, rates will not rise and the option will not be exercised.

What to do?

- Receive fixed on a swap
- Sell Caps / Calls
- Sell FRAs
- Sell Payer Swaptions



Client says: "I think the implied forward rates are too low!"

What to do?

- Pay fixed on a swap
- Sell Floors / Calls
- Buy FRAs
- Sell Receiver Swaptions

In this case, you are saying that the PV of the implied forward rates is too low. Then that must mean that the PV on the fixed rate is also too low. So you should pay fixed (and receive LIBOR) on a swap, since you are receiving *a higher floating rate* than you think you should be.

You could also consider selling floors (puts). You collect a premium and, if your view is correct, the LIBOR will climb higher than it is expected to and the floor will not be exercised.

You could also buy a Forward Rate Agreement (FRA) and benefit if rates trend above the contract level on settlement.

Finally, you could sell a receiver's swaption, giving a counterparty the right to receive fixed in the future. If you are correct, rates will rise above the market's expectations and the option will not be exercised.

Knowledge Demonstration Opportunity

1) The par yields for $100 notional, quarterly coupon bonds trading around par maturing in 3 months, 6 months, 9 months and 1 year are as below. Plot the spot curve.

Price	Yield	Coupon	Maturity
$100	1.5%	1.5%	3 months
$100	2%	2%	6 months
$100	2.5%	2.5%	9 months
$100	3%	3%	1 year

2) The 2 year spot rate is 3.75%. Compute the 1 year forward rate using the spot curve derived in the previous question.

3) It is the 1st of Jan, 3000 A.D. Jupiter Real Estate Corp (JREC) is issuing 5 year bonds at par, denominated in Jeuros (Jupiter's currency) to finance their new space-acquisition. There has however been a very unstable government in Jupiter due to frequent inter-galactic warfare. S&P have issued a BB credit rating for JREC. The bonds are also call-able and the cost of the call option is 80bp. US Treasuries are now rated A, while Saturn Treasuries (STs) are AAA rated. Using the tables below determine an appropriate spread to be added over STs to price Jupiter Bonds. All bonds trade at par. (Average spread over Jupiter Treasuries (JTs) for the Jupiter Real Estate Bond sector is 0.7%)

Ticker	Currency	Structure	Maturity	YTM	Rating
US T-bond	USD	BULLET	5 years	4.4%	A
Saturn ST-bond	USD	BULLET	5 years	3.7%	AAA
US T-bill	USD	BULLET	3 months	1.3%	A
Jupiter JT-bill	USD	BULLET	3 months	2.4%	B
Jupiter JT-bill	Jeuro	BULLET	3 months	2.6%	B

Answers

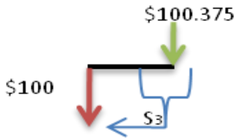
1) Here is the par yield curve as given—

Price	Yield	Coupon	Maturity
$100	1.5%	1.5%	3 months
$100	2%	2%	6 months
$100	2.5%	2.5%	9 months
$100	3%	3%	1 year

The spot curve is computed by bootstrapping as follows—

1

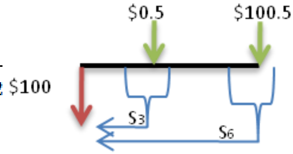
$$100 = \frac{100.375}{1+\frac{S3}{4}}$$



$S_3 = 1.5\%$

2

$$100 = \frac{0.5}{1+\frac{0.015}{4}} + \frac{100.5}{(1+\frac{S6}{4})^2}$$

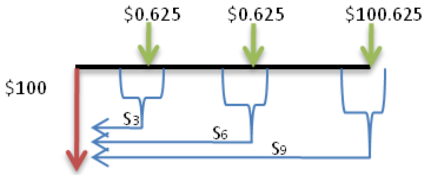


$S_6 = 2.0046\%$

3

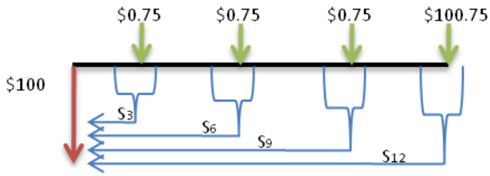
$$100 = \frac{0.625}{(1+\frac{0.015}{4})} + \frac{0.625}{(1+\frac{0.020046}{4})^2} + \frac{100.625}{(1+\frac{S9}{4})^3}$$

$S_9 = 2.50417\%$

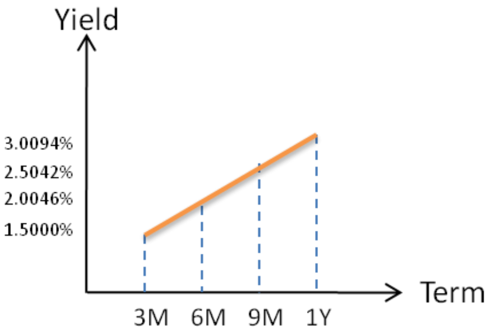


4

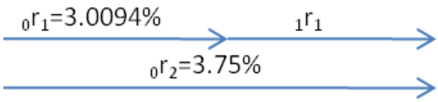
$$100 = \frac{0.75}{(1+\frac{0.015}{4})} + \frac{0.75}{(1+\frac{0.020046}{4})^2} + \frac{0.75}{(1+\frac{0.0250417}{4})^3} + \frac{100.75}{(1+\frac{S12}{4})^4}$$



$S_{12} = 3.0094\%$



2) One year spot rate = 3.0094%, Two year spot rate = 3.75%



$(1+0.030094)(1+{}_1r_1) = (1+0.0375)^2$
Solving, we get $_1r_1 = 4.4959\%$

3)
Credit Spread between T-bill and JT-bill = 2.4%-1.3% = 1.1%
Liquidity spread between USD denominated and Jeuro denominated JT-bill = 2.6%-2.4% = 0.2%

Saturn 5 year ST-bond	3.7%
Add +0.7% for US T-bond	4.4%
Add +1.1% Credit Spread for JT-bond (as determined above)	5.5%
Add +0.2% Liquidity for Jeuro denomination	5.7%
Add +0.7% spread over JT for real estate sector	6.4%
Add +0.8% (80bp) for embedded call option	7.2%

Total spread over STs = (7.2%-3.7%) = 3.5%

Notes

Notes

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